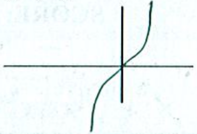
Rewrite $\operatorname{sech}\left(\frac{1}{2}\ln 3\right)$ in terms of exponential functions and simplify. **SCORE: ______ / 3 PTS**

$$\frac{2}{e^{\frac{1}{2}\ln 3}+e^{-\frac{1}{2}\ln 3}} = \frac{2}{e^{\ln 3^{\frac{1}{2}}}+e^{\ln 3^{-\frac{1}{2}}}} = \frac{2}{\sqrt{3}+\frac{1}{\sqrt{3}}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3+1} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

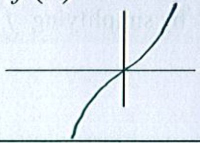
(1) (1) (½) (½)

Sketch the general shape and position of the following graphs. GRADED BY ME **SCORE: ______ / 3 PTS**
(Don't worry about specific $x-$ or $y-$ coordinates.)

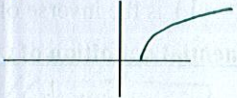
$f(x) = \tanh^{-1} x$



$f(x) = \sinh x$



$f(x) = \cosh^{-1} x$



Write and **prove** a formula for $\sinh(x+y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$. **SCORE: ______ / 6 PTS**

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y \quad (1)$$

$$= \frac{e^x - e^{-x}}{2} \frac{e^y + e^{-y}}{2} + \frac{e^x + e^{-x}}{2} \frac{e^y - e^{-y}}{2} \quad (1)$$

$$= \frac{(e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}) + (e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y})}{4} \quad (1)\ (1)\ (\tfrac{1}{2})$$

$$= \frac{2e^{x+y} - 2e^{-x-y}}{4} \quad (1)$$

$$= \frac{e^{x+y} - e^{-(x+y)}}{2} \quad (\tfrac{1}{2})$$

Prove that $g(x)=\ln(x+\sqrt{x^2-1})$ is the inverse of $f(x)=\cosh x$ by simplifying $f(g(x))$. **SCORE: ______ / 5 PTS**

**You may need to use the exponential definition of $\cosh x$.**

$$\cosh\left(\ln\left(x+\sqrt{x^2-1}\right)\right)$$

$$=\frac{e^{\ln(x+\sqrt{x^2-1})}+e^{-\ln(x+\sqrt{x^2-1})}}{2} \quad (1)$$

$$=\frac{x+\sqrt{x^2-1}+\frac{1}{x+\sqrt{x^2-1}}}{2} \quad (1) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$$

$$=\frac{(x+\sqrt{x^2-1})^2+1}{2(x+\sqrt{x^2-1})}$$

$$=\frac{x^2+2x\sqrt{x^2-1}+x^2-1+1}{2(x+\sqrt{x^2-1})} \quad (1)$$

$$=\frac{2x^2+2x\sqrt{x^2-1}}{2(x+\sqrt{x^2-1})}$$

$$=\frac{2x(x+\sqrt{x^2-1})}{2(x+\sqrt{x^2-1})} \quad (1)$$

$$= x \quad (1)$$

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry. **SCORE: ______ / 7 PTS**

[a] Write that identity involving $\sinh x$ and $\cosh x$. **<u>You do NOT need to prove the identity.</u>**

$$\cosh^2 x - \sinh^2 x = 1 \quad (1)$$

[b] Write the identity for $\cosh 2x$ that uses both $\sinh x$ and $\cosh x$ simultaneously. **<u>You do NOT need to prove the identity.</u>**

$$\cosh 2x = \cosh^2 x + \sinh^2 x \quad (1)$$

[c] Use the results of [a] and [b] to find and **<u>prove</u>** an identity for $\cosh 2x$ that uses only $\sinh x$.

$$\cosh 2x = (1 + \sinh^2 x) + \sinh^2 x \quad (1)$$

$$= 1 + 2\sinh^2 x \quad (\tfrac{1}{2})$$

[d] If $\tanh x = -\frac{2}{3}$, find $\sinh x$ using identities.

**<u>You must explicitly show the use of the identities but you do NOT need to prove the identities.</u>**
**<u>Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.</u>**

$$\operatorname{sech}^2 x = 1 - \tanh^2 x$$

$$= 1 - \frac{4}{9} \quad (1)$$

$$= \frac{5}{9}$$

$$\operatorname{sech} x = \frac{\sqrt{5}}{3} \quad (\tfrac{1}{2})$$

SINCE $\operatorname{sech} x > 0$ FOR ALL $x \in \mathbb{R}$

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}} \quad (\tfrac{1}{2})$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

SO $\sinh x = \tanh x \cosh x$

$$= -\frac{2}{3} \cdot \frac{3}{\sqrt{5}} \quad (1)$$

$$= -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5} \quad (\tfrac{1}{2})$$